



Institute for Empirical Research in Economics
University of Zurich

Working Paper Series
ISSN 1424-0459

Working Paper No. 229

On the Notion of the First Best in Standard Hidden Action Problems

Christian Ewerhart and Christoph Nitzsche

February 2005

On the Notion of the First Best in Standard Hidden Action Problems*

Christian Ewerhart[†]

Christoph Nitzsche[‡]

December 2004

Summary. It is well known that ex-ante randomization can improve upon second best contracts in principal-agent problems. In this note, we show that even the first-best can be dominated by a random contract. Our example is cast in a standard textbook set-up with two effort levels and two states of nature.

Keywords and Phrases. Hidden action, first-best, ex-ante randomization.

JEL Classification Numbers. D82.

*We are indebted to Urs Schweizer for helpful discussions.

[†]University of Zurich, Institute for Empirical Research in Economics (IEW), Winterthurerstrasse 30, CH-8006 Zurich, Switzerland; e-mail: christian.ewerhart@iew.unizh.ch

[‡]University of Zurich, Institute for Empirical Research in Economics (IEW), Winterthurerstrasse 30, CH-8006 Zurich, Switzerland; e-mail: nitzsche@iew.unizh.ch

1 Introduction

The analysis of problems involving hidden action usually starts by characterizing the first best solution, which is understood as being equivalent to unconstrained Pareto optimality.¹ The typical first best contract commits the agent to a specific behavior by conditioning any favorable payment solely to this action. If the principal is risk-neutral and the agent is risk-averse, it is well known that such a contract entails a constant payment on the part of the principal. Since complete insurance is what economists would expect if there were a market where all contingencies could be traded this notion of first-best seems to be highly intuitive. However, we shall give an example that shows that even the first-best can be improved.

Our example builds on the possibility of randomizing over contracts. The underlying idea is not new and was recognized by Baiman [2]. He pointed out that any pure contract may be suboptimal as soon as randomization over contracts is admitted. This issue has been investigated by various authors for the case of moral hazard (see Gjesdal [4], Grossman and Hart [5], Arnott and Stiglitz [1]). However, to our knowledge, under the heading *ex-ante randomization* Fellingham, Kwon and Newman [3] were the first to give a formal discussion of the desirability of randomization in the full informa-

¹Shavell [7] and Grossmann and Hart [5] defined a Pareto efficient allocation as a fee schedule and an effort level that maximizes the principal's utility subject to the agent's utility being equal to some outside option. In Ross [6], Pareto efficiency requires a weighted sum of the agent's and the principal's utility to be maximized. However, Ross did not consider utility functions with the action taken as a direct argument.

tion case, i.e. situations with no informational asymmetries such as settings that would lead to the first best contract. They formulated sufficient conditions that render ex-ante randomization efficient. However, their analysis is restricted to local non-convexities around the first-best. In contrast, as our example shows, ex-ante randomization can be profitable even with concave preferences and expected payoffs, thus implying that efficient ex-ante randomization could not be ruled out in general if the latter two were concave. The result highlights the importance of global properties of the Pareto frontier over local ones.

2 The Example

An agent chooses between two legally enforceable effort levels $q \in \{0, 1\}$. The principal pays a wage $\omega \geq 0$. Assume that the agent's utility is given by the function $U(\omega, q) = V(\omega) - C(q)$, where $V(\omega) = \ln(\omega + 1)$ stands for the utility resulting from a wage ω and $C(0) = 0, C(1) = \ln 2$ are the costs of the respective effort levels. If the agent does not accept the contract his outside option is $U_0 = 0$. The principal receives a payoff $\Pi(\pi, \omega) = \pi - \omega$, where π is random with conditional expectation values $\mathbb{E}[\pi|q = 0] = 0$ and $\mathbb{E}[\pi|q = 1] = 1 + \varepsilon$ for some constant $\varepsilon > 0$. It is straightforward to verify that the first-best contract in this simple setting is $(w^*, q^*) = (1, 1)$. Then, for the agent, there results a utility equal to his outside option, whereas the principal would make a profit of $\mathbb{E}[\Pi(1, 1)] = \varepsilon > 0$.

Now suppose that the principal has access to a publicly observable universal random generator, e.g. a dice, so that the principal could offer contracts of the form (ω, p) where p depicts the probability of having the agent committed to the effort level $q = 1$. For such a contract to be accepted by the agent, it must hold that

$$V(\omega) - (1 - p) C(0) - p C(1) \geq 0$$

which makes the wage depend on p , namely $\omega = \omega(p)$ or, more precisely, $\omega(p) = 2^p - 1$. The principal's payoff would be

$$\begin{aligned} \mathbb{E}_p[\Pi] &= (1 - p) \mathbb{E}[\pi|q = 0] + p \mathbb{E}[\pi|q = 1] - \omega(p) \\ &= p (1 + \varepsilon) - 2^p + 1. \end{aligned}$$

The first order condition for the associated maximization problem (for ε sufficiently small) is

$$\frac{\partial \mathbb{E}_p[\Pi]}{\partial p} = 1 + \varepsilon - \ln 2 \cdot 2^{p^*} = 0 \quad \text{or} \quad p^* = \frac{\ln(1 + \varepsilon) - \ln(\ln 2)}{\ln 2}.$$

Note that

$$\lim_{\varepsilon \rightarrow 0} p^* = \frac{\ln(\ln 2)}{\ln 2} \approx 0.529$$

and $\lim_{\varepsilon \rightarrow 0} \mathbb{E}_{p^*}[\Pi] \approx 0.086$.

In sum, while the principal always prefers a contract with $q = 1$ to a

contract with $q = 0$, he would like to have the agent working only with a certain probability, provided this sort of contract is feasible. In particular, for a sufficiently small ε , such a contract always strictly dominates the first-best solution $(1, 1)$. The situation is depicted in Figure 1. Each curve represents the set of utility pairs that result from different allocations between the principal and the agent, given an effort level. The first-best leads to an utility pair of A , whereas prior randomization would result in an expected profit given by B . After randomization has taken place the resulting allocations could C or D , respectively.

- place

Figure 1

here -

3 Discussion

Suppose a higher effort level is associated with a higher expected payoff. Then, the intuition in our example works as follows: Let (ω_1, q_1) and (ω_2, q_2) be wage-effort combination with $q_1 = q^*$, $q_2 < q^*$ and $\omega_1 < \omega^*$, $\omega_2 = \omega^*$.² The agent is offered these two contracts in form of a lottery with probability p and $(1 - p)$, respectively, such that his participation constraint still holds. If he is risk averse in income the agent would be willing to forego some of his income ω^* in favor of a reduced risk exposure, which, in turn, causes the principal's profit to rise. Hence, on one hand, a prior randomization between two contracts allows the principal to push the expected wage under its first best level ω^* . However, on the other hand, the induced randomization over

²Note that $U(\omega_1, q_1) < U(\omega^*, q^*) < U(\omega_2, q_2)$.

effort levels might lower the expected payoff. Now, if the gain in the expected profit due to the wage cut outweighs the potential loss due to a decreased expected payoff, a new contract can be written that is favorable to both sides.

The assumption of a finite effort set is not crucial for our argument. Moreover, a necessary condition for ex-ante randomization to improve upon the first-best is that the Pareto frontier is not concave. In the case of continuous effort levels this includes non-concave utility functions or increasing return to scales as it was studied by Fellingham et al. However, as the example demonstrates, the Pareto frontier might well be concave around the first-best but, nevertheless, admit profitable ex-ante randomization.

More general, the result is due to the particular relationship between the utility function over wealth V , the cost function C and the conditional expected payoff $\mathbb{E}[\pi|q]$. In the case of separable utility functions, at the beginning, i.e. q small, costs attributed to a higher effort level must increase substantially compared to the increase in the expected payoff whereas, at the end, i.e. q large, the relation must be reversed.³ However, the result does not depend on the assumption of separability of the agent's utility function U as long as the agent is risk-averse in wealth and U decreases in effort. Respective examples can easily be constructed. According to the last paragraph, just let $U(\omega, q)$ be bounded. Then, if for some $\bar{q} > q^*$, it holds that $U(0, \bar{q}) < -\alpha$ and $\mathbb{E}[\pi|\bar{q}]$ sufficiently large compared to $\mathbb{E}[\pi|q^*]$ a similar result can be achieved.

It might be worthwhile pointing out that optimal ex-ante randomization

³In the discrete case only the first condition is needed.

(in the case of a discrete choice set) may involve the use of those pure effort levels which the principal would never be able to induce the agent to willingly choose, for it were too costly for the principal to do so. To see this, suppose that the agent's income utility V is bounded from above, by some $\alpha > 0$. Now, if $C(1) > \alpha$, there is no contract committing the agent to $q = 1$ and, hence, the optimal contract would be at $q = 0$. In contrast, if the principal is able to write contracts in the sense of our example, the expected payoff of both parties could be increased by conditioning partly on $q = 1$.

References

- [1] Arnott, R. J. and Stiglitz, J. E. Randomization with asymmetric information. *RAND Journal of Economics*, **19**(3), 344–362 (1988).
- [2] Baiman, S. Agency research in managerial accounting: A survey. *Journal of Accounting Literature*, **1**, 154–213 (1982).
- [3] Fellingham, J. C., Kwon, Y. K., and Newman, D. P. Ex ante randomization in agency models. *RAND Journal of Economics*, **15**(2), 290–301 (1984).
- [4] Gjesdal, F. Information and incentives: The agency information problem. *Review of Economics Studies*, **49**, 373–390 (1982).
- [5] Grossman, S. J. and Hart, O. D. An analysis of the principal-agent problem. *Econometrica*, **51**(1), 7–45 (1983).

- [6] Ross, S. A. The economic theory of agency: The principal's problem. *American Economic Review*, **63**(2), 134–139 (1973).
- [7] Shavell, S. Risk sharing and incentives in the principal and agent relationship. *The Bell Journal of Economics*, **10**(1), 55–73 (1979).
- [8] Strausz, R. Mitigating non-contractibility with interim randomization. *Journal of Institutional and Theoretical Economics*, **157**(2), 231–245 (2001).

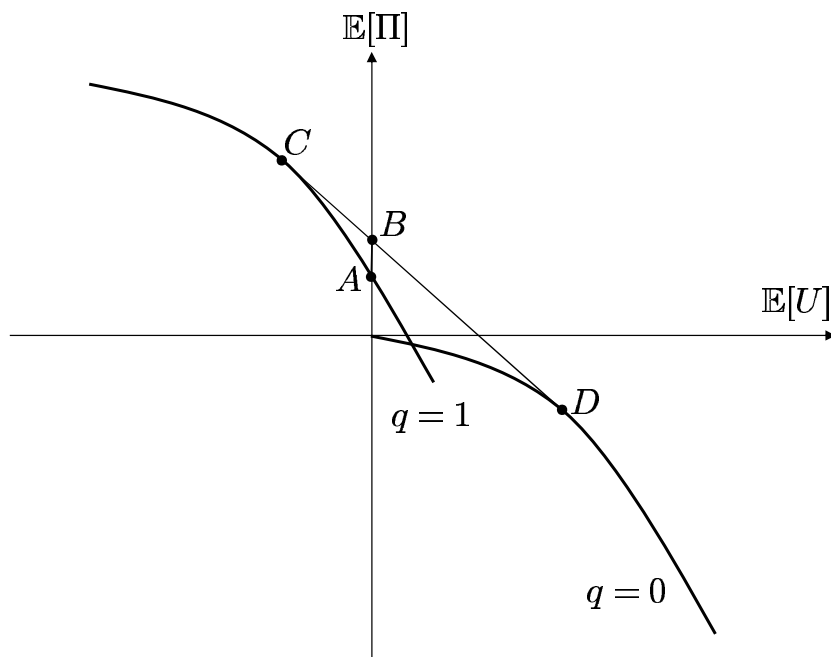


Figure 1: